IMPACT OF CAUSTICS ON ACOUSTICS OF WHISPERING GROTTOES IN THE OLIWA PARK IN GDAŃSK

1. INTRODUCTION

In large historical interiors formed by curved surfaces, it is very likely to come across acoustical singularities consisting in "sliding" of sound along a concave wall or concentration of sound in a distant location of the room. The physical nature of these singularities may be different but the effect is the same – two people can hear each other over a large distance without using any additional sound amplification systems.

In contemporary auditoriums, these phenomena are considered an acoustic flaw. However, the present study deals with rooms created at a time when room acoustics were not yet a scientific discipline in today's sense. The discussed singularities were therefore not subject to acoustic correction, and even their existence was not always noticed. They can be mostly found in historical premises serving stately, ceremonial, liturgical, or similar purposes where these effects usually were detected accidentally, sometimes many years after erection of the building. Descriptions of these places of interest can be found in books published as early as in the beginnings of development of the architectural acoustics as a scientific discipline [13, 14, 16, 17].

The aim of this paper is to prove that the discussed phenomena occur as a result of formation of a caustic, i.e. the envelope of a bundle of sound rays originated from a point source and reflected by a concave surface. The paper discusses in detail the 18th-century historic object, which has been built from scratch to demonstrate the whispering wall phenomenon to the public, with deliberate use the caustic for this purpose.

2. CAUSTICS IN ROOM ACOUSTICS

Figure 1 shows the formation of caustics based on the principle of geometric optics. This principle is also used in room acoustics, where discrete sound rays are interpreted as the directions of propagation of waves in the room [10].

Geometrical structure of the acoustic field in rooms dominates energetically its other structures of the field, i.e. the wave structure and the statistical structure. In particular, this refers to the initial phase of formation of acoustic field in a room, typically of a duration of several dozen milliseconds. Formation of caustics in this very period of time may therefore have a considerable effect on the acoustics of a given room. Generally, this is an adverse effect. However, as it is shown in Figure 2, an already existing caustic may be utilised to the benefit of acoustics of

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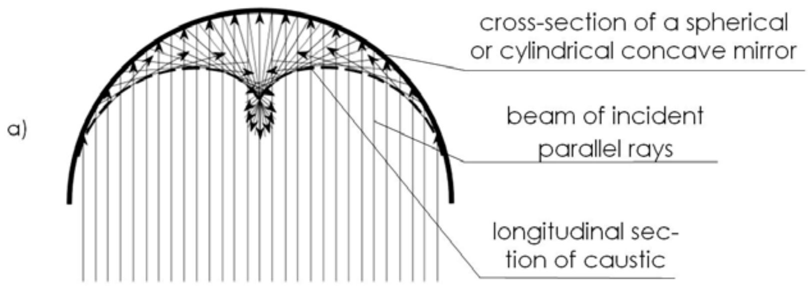
the interior. In Chapter 3 it is also demonstrated that in historical premises caustic is sometimes deliberately formed by the architect.

Historic interiors are frequently formed by vaults, walls build on a curvilinear plans or comprise other large scale elements that represent curved surfaces. In such rooms, there is a natural tendency to concentrate the sound. Usually it is assumed that such concentrations take the form of a point-like focus or some region of space. A point-like focus occurs only when the mirror is a sector of a paraboloid and when the sound source is situated on its geometric axis. In real rooms, these conditions are usually not met. Curvatures of walls and vaults are designed according to construction requirements and may take the form of a paraboloid sector only by chance. Moreover, moving sound sources, such as people or musical instruments, can find themselves on symmetry axes of these curved features only accidentally. For these reasons, sound energy concentrations in rooms take typically the form of a caustic.



b)

Fig. 1. Caustic as a manifestation of spherical aberration of spherical or cylindrical concave mirror: (a) geometrical and (b) physical counterpart of the caustic [7]

Figure 2 shows the Mormon Tabernacle in Salt Lake City, USA, as an example of using an existing caustic to improve audibility in a part of the hall most distant from the speaker. In this case, the caustic is a result of the sound reflection from the rear portion of the ceiling constituting a concave acoustic mirror. Before the balcony was constructed, the sound energy focused by the mirror fell over a large portion of the auditorium area which resulted in low sound intensity. Once the balcony of suitable height was added, the same flux of acoustic energy falls onto a significantly smaller surface area. This improved audibility on the balcony [4].

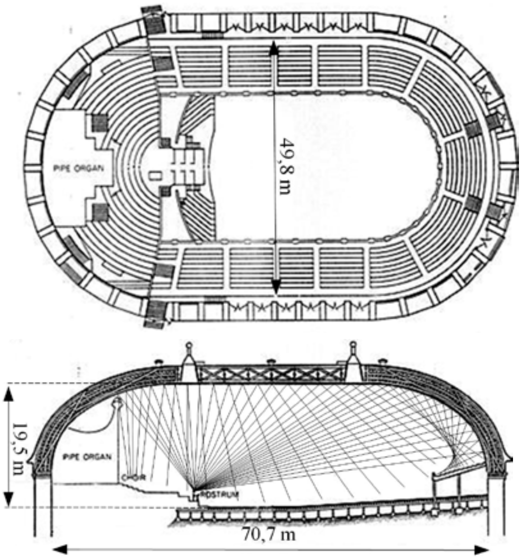


Fig. 2. The Mormon Tabernacle, Salt Lake City, Utah, USA (1869). The lower drawing shows a caustic over the balcony in the rear portion of the hall [6, 11]

3. PHENOMENON OF DIFFRACTION IN THE

Chapter 2 contains a simplification, resulting from the assumptions of the geometric model of the acoustic field. In this model, the field is composed of discrete sound rays and an empty space between them. In this field, the phenomenon of diffraction is neglected. In contrast to this approach, the physical acoustic field is a *continuum* formed by moving fronts of the waves. An important element of the physical acoustic field is the phenomenon of diffraction. The structure of physical field, including a phenomenon of diffraction, depends on the wavelength. One of the effects of diffraction in rooms is the ever-weaker "recognition" of the geometric details of the obstacle reflecting the sound, along with the increase of wavelength. This means that long enough waves do not "see" the geometric details of the obstacle and reflect from of it like from a flat surface. The relationship between the dimensions of the obstacle and the size of its elements that reflect the wave in a mirror manner, i.e. the details of its surface, depth of the curvature of the obstacle etc., is given in the eqn. (1) [8]:

$$l \geq K\,\lambda \tag{1}$$

where: λ – the length of the longest reflected wave reflected in a mirror manner,
l – the smallest dimension of the obstacle or the size of its detail,
K – factor dependent on the assumed ratio between mirrored or diffused energy.

In the literature, the value of K is generally larger than 1. This means that for a reflection recognized as a mirror one, the obstacle with its geometric details should be greater than the wavelength – according to some authors, up to 4 times [3, 12]. At the smallest dimension of a sound reflecting element of e.g. 1.5 m, the obstacle acts then as a the mirror for waves of frequencies greater than 230-900 Hz. Also, *K* values less than 1 can be found, where reflection is still considered to be a mirror one, with much larger component of the scattered energy (e.g. *K* = 1/3 [15]). With such a criterion, a 1.5 m element acts as a mirror for waves of frequencies greater than 75 Hz.

Apart from the phenomenon of diffraction, simplification related to the geometric model of sound field also concerns the phenomenon of sound focusing. As in this model the focuses are represented by points, and the cross sections of the caustics by the lines, their geometric dimension is zero. In the physical acoustic field, the focuses and caustics are the result of the superposition of wavefronts occurring in the area of size depending on the wavelength [9]. This means that for a given spectral composition of sound, the focus takes the form of multiple overlapping areas corresponding to the individual spectral components of the sound. The answer to the question as to the size of the focus area or the "thickness" of cross section of caustics goes beyond the scope of this study. For this reason, the further part of the work is still based on the geometric model of sound field.

4. A CASE STUDY – THE WHISPERING GROTTOES IN GDAŃSK'S OLIWA PARK

4.1. DESCRIPTION OF THE STRUCTURE

The Whispering Grottoes belong to a group of well-preserved components of the eighteenth-century landscape park in Gdańsk-Oliwa. The structure comprises two symmetrical oval grottoes built opposite each other, meant as an attraction serving outdoor demonstration of the laws of acoustics (Fig. 3). If two persons stand in the grottoes back to back, they are able to hear each other much better than in open space. Before starting the renovation project covering the grottoes and a park alley between them, it become necessary to check whether this effect will reappear after restoration. The present paper is an elaboration of the expert's opinion aimed at providing an answer to this question.

Fig. 3. The Whispering Grottoes in Oliwa Park, Gdańsk (source: photo by T. Lepert-Trude [1])

Each of the grottoes is composed of a ¼ of a sphere constituting an acoustic mirror and two segments of cylindrical surfaces serving as a structural extension of the mirror. One segment can be found between the spherical part of the grotto and the ground, while the second one constitutes the entrance to the grotto (Fig. 4). The spherical sections are slightly distorted — in the vertical and horizontal section, the radii of curvature of acoustic mirrors are 142.5 cm and 130 cm respectively (dashed and solid lines in Fig. 4).

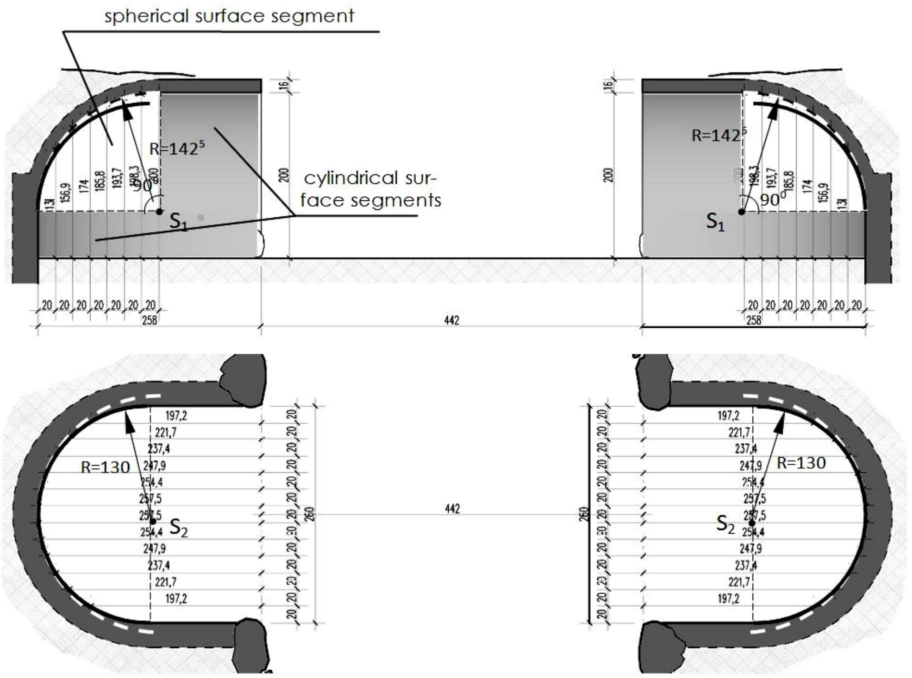


Fig. 4. Vertical and horizontal section of the Whispering Grottoes in Gdańsk's Oliwa Park. The dashed line and the solid line mark curvatures of the mirror in vertical and horizontal section, respectively, with S_1 and S_2 marking centres of curvature ($S_1 \neq S_2$) (source: own study)

4.2. The mechanism of sound amplification in the grottoes

The Whispering Grottoes represent a system of two concave spherical mirrors situated opposite each other. Rays incoming from the emitting mirror are focused in the receiving mirror, what creates the effect of amplification (Fig. 5). As can be seen from Figure 1, this effect is a result of spherical aberration typical for concave mirrors.

In case of two spherical mirrors, apart from the caustic formed by rays incoming to the receiving mirror, it is also possible to identify a caustic formed in the emitting mirror. For the purpose of the present study, they will be referred to as the receiving caustic and the emitting caustic. On the analogy to the receiving caustic, the emitting caustic is an envelope of rays, emitted by isotropic sources with definite locations. After the reflection, these rays form a bundle of parallel rays. "Definite locations" means the geometric locus of potential locations of such sources (Fig. 5a). The emitting caustic and the receiving caustic are mirror reflections of one another.

Aurally perceptible amplification of sound occurs when the speaker and the listener are situated on symmetrical fragments of the emitting and receiving caustic, respectively (Fig. 6a, b and Fig. 7). When the head of either the speaker or the listener is positioned outside their respective caustic, the sound amplification effect disappears (Fig. 6c).

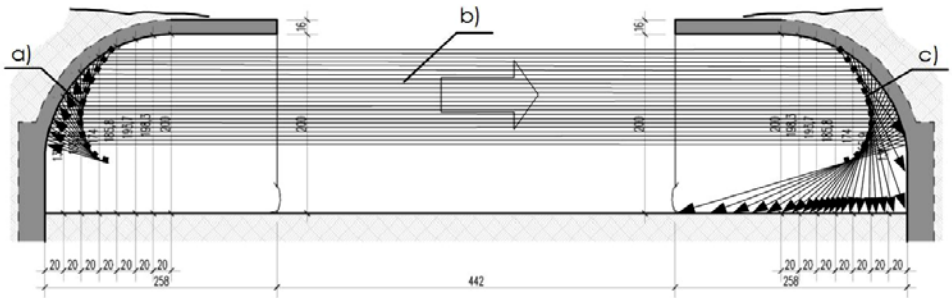


Fig. 5. Vertical section of the Whispering Grottoes in Oliwa Park: (a) the emitting caustic, i.e. geometric locus of isotropic sources of rays; (b) the bundle or parallel rays; (c) the receiving caustic, i.e. the region where the parallel rays focus after reflection (source: own study)

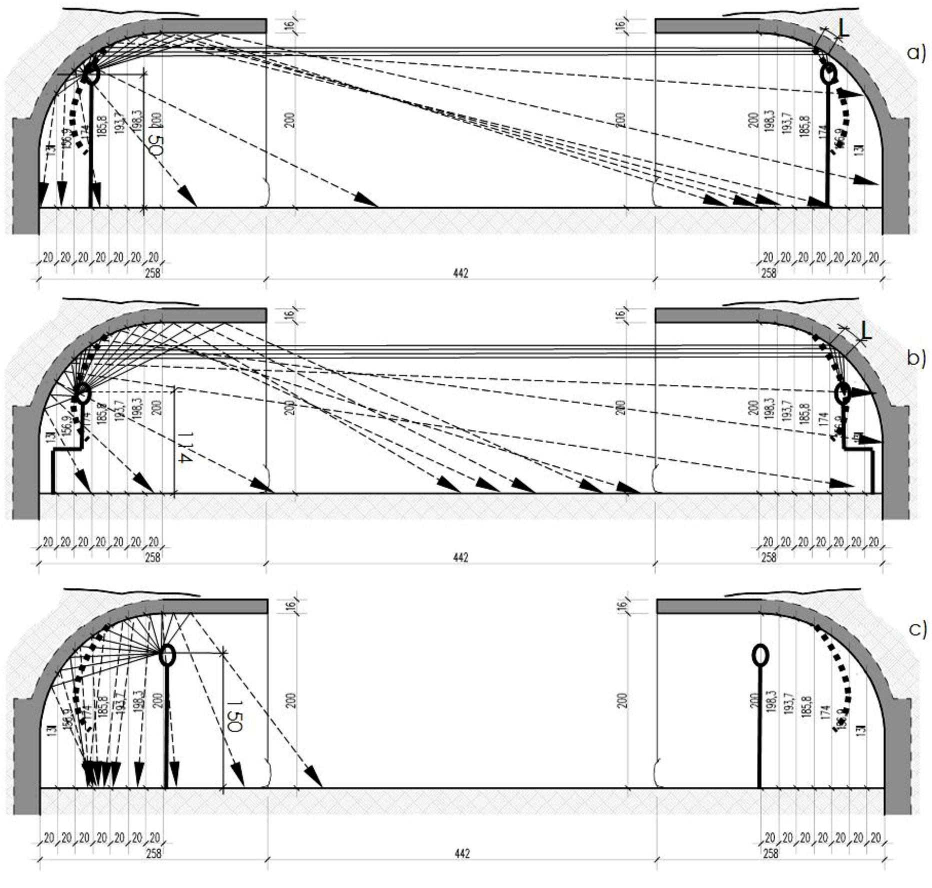


Fig. 6. Sound rays in the vertical section of Whispering Grottoes at the present ground level. "L" marks the relevant sound-focusing segment of the acoustic mirror. The lips of a person about 166-168 cm high are situated at a height of about 150 cm (source: own study)

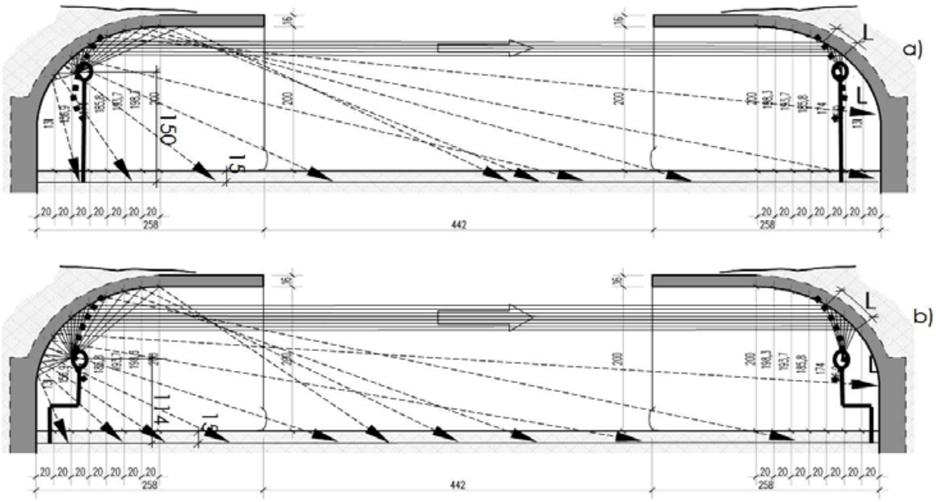


Fig. 7. Sound rays in the vertical section of the Whispering Grottoes in Oliwa Park for (a) standing and (b) sitting position of both speaker and listener. Sound focusing occurs with higher intensity with the ground level lowered by 15 cm – the length of segment "L" increases in comparison with this of Figure 6a and b (source: own study)

As the energy of focused sound is stretched out in the form of the caustic, the sound amplification effect becomes weaker, because only a portion of the mirrors surface takes part in sound focusing. The size of the sound focusing surface depends on position of speaker and listener on the caustic (see segments denoted "L" in Fig. 6a, b and Fig. 7). With increasing area of this surface, the sound amplification level also increases and therefore it is possible to determine such position of the speaker and the listener, for which the sound amplification effect reaches its maximum. This position corresponds to the caustic edge, where concentration of rays forming the caustic is strongest. This follows from both geometrical analysis of the caustic forming process (Fig. 1a, Fig. 6c) and experimental studies concerning technical applications of spherical mirrors [2].

In Figures 6 and 7, dashed lines represent rays which do not take part in formation of the discussed acoustic curiosity of the grottoes.

4.3. Heritage preservation determinants

The caustic as a form of focus stretch-out in concave optical mirrors is known for several centuries now ("One of the earliest discoveries in optics (F. Maurolycus, 1575) was that the rays of a normal system are tangential to a surface, the so-called caustic surface" [5]). However, despite the relationships between the room acoustics and the principles of geometric optics, the term "caustics" is in principle absent in the contemporary literature on room acoustics.

Taking into account positions of caustics in relation to inner surfaces of the grottoes, it must be assumed that the designer of the grottoes considered the ergonomic aspect of the structures. He had to provide that a visitor could place his/her mouth in proper location and then, taking into account the average height of a person, adopt an appropriate level of ground in the grottoes. This proves that the eighteenth-century architect designing the Whispering Grottoes in Oliwa Park was undoubtedly familiar with the rule of origination of caustics and used the knowledge consciously. The designer for the whole park landscaping, dated for the 2nd half of the eighteenth century, was Kazimierz Dębiński of Kock [18]. The name of the designer of the grottoes themselves remains unknown.

At the present level of ground in the grottoes, the sound amplification effect can be observed when the experiment is carried out by persons about 165-170 cm tall, but only keeping an uncomfortable position with their faces a few cm from the grotto wall (Fig. 6a). One should therefore assume that the original ground level in the grottoes was lower. The ground level lowering by about 25-30 cm allows both the persons to move away from the grotto wall by a distance of about 30 cm and still remain within the caustic area.

Therefore, the original gradient of the paths leading from the park to the Whispering Grottoes was probably much larger than the present one and can be estimated at about 15%. Bearing in mind the current regulations due to the heritage conservation, which limit the gradient of park alleys to 6%, it is assumed that in the course of the planned restoration of the grottoes, the ground level will be lowered only by about 15 cm (Fig. 7). It can be expected that this will be sufficient to enhance the specific acoustic effect of the grottoes. This is a result of increasing sound-focusing surface area of the mirror at the lower position of the observer (cf. segment "L" in Fig. 6a, b and Fig. 7).

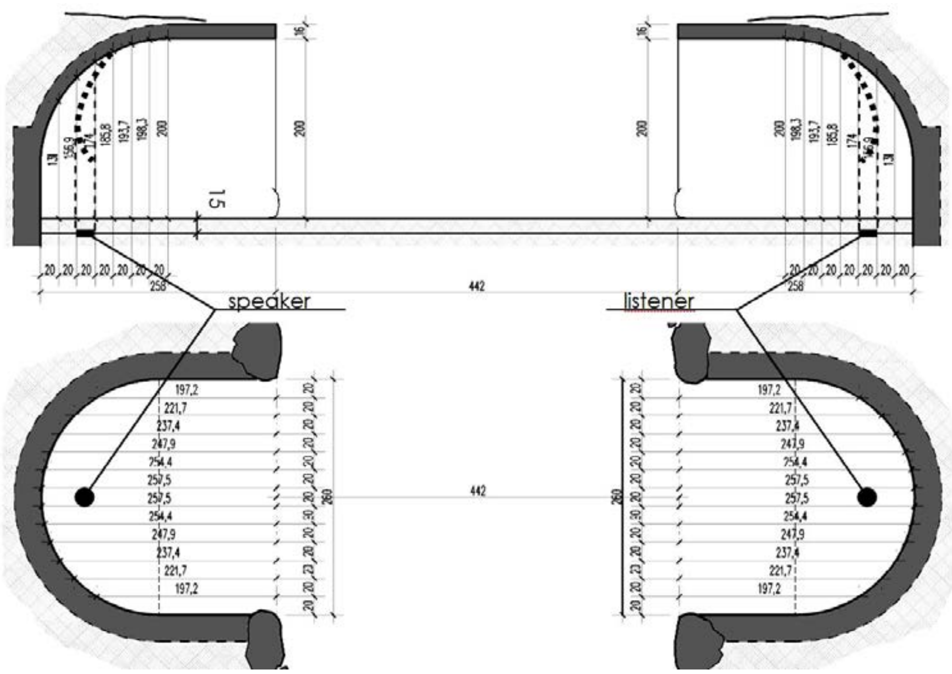


Fig. 8. Correct positions of the speaker and the listener on the grottoes vertical and horizontal section (source: own study)

The shape of the grottoes requires that both the speaker and the listener stand right in front of and facing the wall, which is neither obvious nor intuitive. To ensure correct execution of the experiment, it is then useful to mark positions which should be taken by the two persons, e.g. by using a distinguishing material to pave these specific places on the ground of grottoes. The areas correspond to vertical projections of the caustics onto the ground plane (Fig. 8). The planned scope of restoration work includes also replacing the existing asphalt pavement with a gravel surface. The type of the material used to pave the surface of paths and the ground inside the grottoes is of no significance for the sound amplification effect as the bundle of parallel rays propagates between the grottoes about 0.8 m to 1.8 m over the ground level.

To remove saline spots, pieces of peeling plaster, and marks of repairs made on soffits of the grottoes in the past, it is planned that smooth putties applied on them currently will be replaced with a special material used for renovation purposes. For acoustical reasons, it is better to use a cohesive plaster, as before reaching the listener the sound is reflected first in the emitting and then in the receiving grotto. To maintain the sound amplification effect, any porous and therefore sound absorbing material should be avoided. However, the stoppage of progressing erosion of the grottoes construction is considered a higher-priority issue, so it is accepted that a porous renovation plaster will be used with a coat of silicate paints.

5. CONCLUDING REMARKS

Large-scale arched vaults and concave walls are structural elements typical for historical interiors. Due to their dimensions, these features affect the acoustics of the rooms acting as concave acoustic mirrors and resulting in local concentration of the reflected sound energy. Such concentration rarely takes the form of a point-like focus, contrary to examples most frequently discussed in the literature. Instead, the predominant form of sound concentration is the caustic, barely present in the literature on architectural acoustics. The caustic is a hyper surface being the envelope of a bundle of sound rays reflected from concave mirror.

It is demonstrated in this paper that the concept of caustic, known in optics for a long time, was familiar to architects of historic buildings erected with the intention to demonstrate acoustical curiosities. A structure designed for this purpose, described in detail, are the historic Whispering Grottoes in the eighteenth-century Oliwa Park in Gdańsk. Analysis of geometrical features of the design reveals that the phenomenon of caustic was used intentionally in this case.

Historic sources indicate that acoustical curiosities described in this paper, tempted the architects to design interiors in a way allowing to obtain access to information of confidential or even intimate nature, “for one’s ears only”. In this sense, the paper points out opportunities for widening the scope of studies on the acoustic of historic buildings with their historic context.

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SUMMARY. In the paper, caustics are discussed as ordered forms of focus broadening occurring in concave mirrors. An example of caustics created by the reflection of the light from the concave mirror is shown. Against this background, the possibility to observe caustics in rooms is pointed out. Special emphasis is put on large rooms of historic character, as such interiors frequently include acoustic mirrors in the form of arched vaults and concave walls. As a case study, the eighteenth-century Whispering Grottoes representing one of attractions of the Oliwa Park in Gdańsk was selected, where the phenomenon of forming a caustic was used intentionally to obtain the desired acoustic effect.

Key words: room acoustics, historic interiors, chamber of whispers, blurred focus, caustic