23rd International Conference
ENGINEERING MECHANICS 2017
Svratka, Czech Republic, 15 – 18 May 2017

EXPERIMENTAL VERIFICATION OF VELOCITY DISTRIBUTION IN DIFFERENT CROSS-SECTIONAL VENTILATION DUCTS

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Abstract: *The paper presents experimental verification of theoretical models for the velocity distribution in circular, rectangular, and rounded rectangular cross-sectional ventilation ducts. This verification is based on the knowledge of circular and rectangular cross-section ducts' properties and the new cross-section contains both aforementioned. These studies are aimed at finding ways of reducing the necessary measurements to determine the average flow rate in ventilation air ducts. Due to the size of the ducts it was decided to determine the flow rate based on the integration of flow velocity over the considered cross-section. This method requires knowledge of velocity distribution in the cross section. The approximation of measured actual profile by the basic and modified Prandtl power-law velocity profile was verified.*

Keywords: ventilation duct, power-law velocity profile, rounded rectangular cross-section duct.

1. Introduction

Testing the ventilation system components and their design can be greatly simplified by finding analytical solutions which determine the velocity distribution of air flow in a duct. Mechatronics Group of Mechanical Engineering Faculty UTP in Bydgoszcz in cooperation with Nucair Technologies Sp. z o.o., Solec Kujawski, Poland, conducted research of new type ventilation ducts of rounded rectangular cross-section. This duct type has significant operational advantages, but the aerodynamic phenomena occurring in these ducts have not been studied yet.

2. Analysis of circular ducts

Before an experimental research and theoretical analysis of rounded rectangular ventilation ducts, circular $D = 0.4\,\mathrm{m}$ and rectangular $W = 0{,}5\,\mathrm{m}$, $H = 0.25\,\mathrm{m}$ ventilation ducts were examined. Flow velocity in particular measurement points was measured using HWA anemometer, while in central point was verified using Prandtl tube (also called Pitot static tube). Measurements were performed along symmetry axes with step of 4 mm. Obtained experimental curves were approximated by theoretical curve known as power-law velocity profile expressed as

Basic power-law velocity profile $$V(r) = V_{\mathrm{C}}\left(1-\frac{r}{R}\right)^{\frac{1}{n}} \quad (\mathrm{m/s}) \tag{1}$$

where $V_{\mathrm{C}} = V_{\max}\,(\mathrm{m/s})$ velocity at the centerline where $r = 0$, $r\,(\mathrm{m})$ radius from $r \in [0; R]$, $R\,(\mathrm{m})$ inner radius of circular duct.

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The method of least squares is used to estimate parameters of theoretical model V_C and n. Estimation for the parameters was obtained by minimizing the sum of squared differences between the measured values and the predicted values under the model. Because, first of all, for a rectangular cross-section, obtained solutions were not very good, an attempt to approximate using the modified power-law velocity profile was taken under consideration. Modified power-law velocity profile is expressed as (Peszyński et al., 2016)

Modified power-law velocity profile $$V(r)=V_C\left(1-\left(\frac{r}{R}\right)^2\right)^{\frac{1}{n}} \quad (2)$$

There are two parameters in two equations (1) and (2): velocity in central point V_C and exponent $1/n$. Fig. 1 presents approximation results of measurement using both equations (1) and (2) for circular duct (pipe) of $R = 0.2\,\text{m}$.

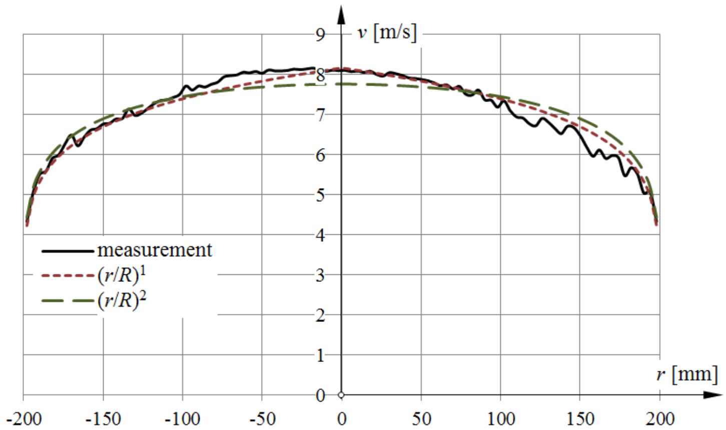

Fig. 1: Measurement results approximated by Prandtl classical and modified power law for $n = 7$.

Notice that approximation by basic Prandtl power-law velocity profile is good. However, this result is not important for practical use. We want to limit the number of measurements. In this case, we should rather seek the optimal exponent $1/n$ for velocity V_C treated as measured parameter. n is integer number in all text books (Cengel et al., 2006; Orzechowski et al., 1997). We must remember that Prandtl power-law velocity profile was invented almost a hundred years ago. In those days, when there was no electronic computer technology, calculations using n as decimal number were almost impossible. Nowadays, it is not a problem. Therefore, velocity V_C is treated as measured constant parameter, and we are looking for decimal number n as approximation parameter. This method has proved itself mainly in the analysis of rectangular ducts.

3. Analysis of rectangular ducts

Application of the Prandtl power law formula is recommended by the literature (Cengel et al., 2006) for analysis of rectangular cross-section ducts. The longer side of the rectangle should be substituted instead of radius, and the degree of its flattening is taken into account by the correction factor.

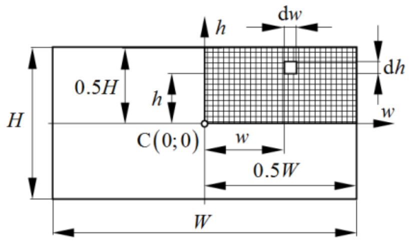

Fig. 2: Basic dimensions for rectangular duct

Another procedure was chosen during presented tests. The velocity distribution along two axes w and h (Fig. 2) was measured. Formulas similar to basic and modified power law were used for approximations, when $r \to w$, and $r \to h$. Then, $\mathrm{d}Q(w,h) = V(w,h)\mathrm{d}A$, $\mathrm{d}A = \mathrm{d}w \cdot \mathrm{d}h$, $Q = \int_A V(w,h)\mathrm{d}A$, and $V_{avg} = Q/A_{\mathrm{rec}}$:

Basic power law $$V(w) = V_{\mathrm{C}}\left(1 - \frac{2w}{W}\right)^{\frac{1}{n}} \quad \text{and} \quad V(w) = V_{\mathrm{C}}\left(1 - \left(\frac{2w}{W}\right)^2\right)^{\frac{1}{n}} \tag{3}$$

Modified power law $$V(h) = V_{\mathrm{C}}\left(1 - \frac{2h}{H}\right)^{\frac{1}{n}} \quad \text{and} \quad V(h) = V_{\mathrm{C}}\left(1 - \left(\frac{2h}{H}\right)^2\right)^{\frac{1}{n}} \tag{4}$$

Fig. 3 shows measurement results and approximation curves for rectangular duct of width $W = 0.5\,\mathrm{m}$ and height $H = 0.25\,\mathrm{m}$, when $V_{\mathrm{C}} = V_{w=0} = V_{h=0} = 7{,}067\,\mathrm{m/s}$. Approximating curves were plotted for $n_1 = 8.45$ in case of basic power law and for $n_1 = 5.67$ in case modified one.

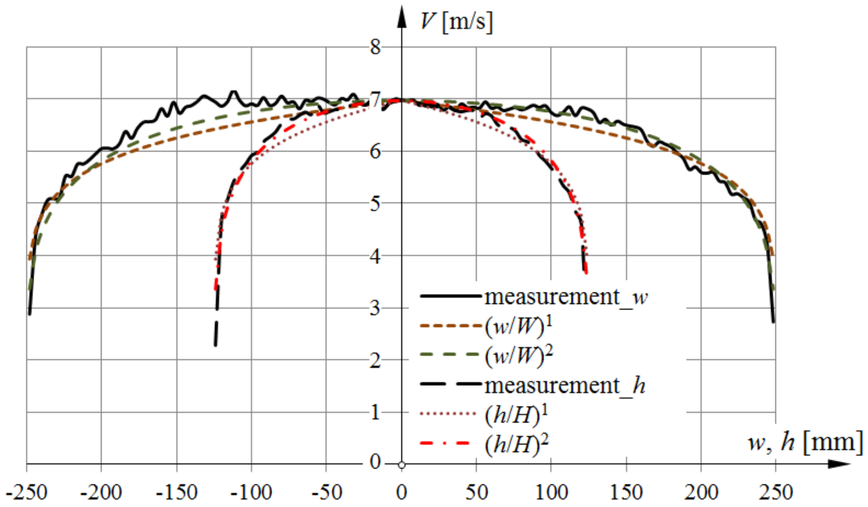

Fig. 3: Measurement results of rectangular duct approximated curves for measured $V_{\mathrm{C}} = 7.067\,\mathrm{m/s}$.

These formulas were verified by numerical verification (Peszyński et al., 2016), the same formulas were used for rounded rectangular ducts.

4. Analysis of rounded rectangular ducts

Rounded rectangular duct is defined by three basic dimensions W, H, R, see Fig. 4.

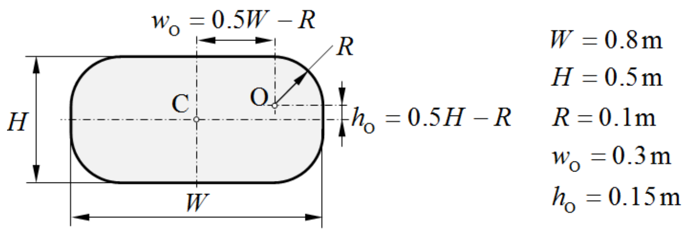

Fig. 4: Tested rounded rectangular duct dimensions.

Fig. 5 shows measurement results for the rounded rectangular duct of width $W = 0.8\,\mathrm{m}$, height $H = 0.5\,\mathrm{m}$, and $R = 0.1\,\mathrm{m}$. These dimensions (Fig. 4) and results (Fig.5) are one of 77 possibilities. Area, perimeter, and hydraulic diameter are $A = 0.391\,\mathrm{m}$, $U = 2.428\,\mathrm{m}$, and $D_{\mathrm{h}} = 0{,}645\,\mathrm{m}$ respectively for this cross-section dimensions. All measurements were provided for $Re = 100\,000$. This Reynolds number implicates that average velocity is $V_{\mathrm{avg}} = 2.055\,\mathrm{m/s}$. Then, by using coefficient of profile fill $\varpi = 0.86$, we can obtain velocity $V_{\mathrm{C}} = V_{\mathrm{avg}}/\varpi = 2{,}389\,\mathrm{m/s}$ in central point C.

The profiles were measured on symmetry axes, w and h. Only $V(w)$ is presented in Fig. 5. Note that V starts from $V = 1.6\,\mathrm{m/s}$ and big fluctuation of $V(w)$ can be observed.

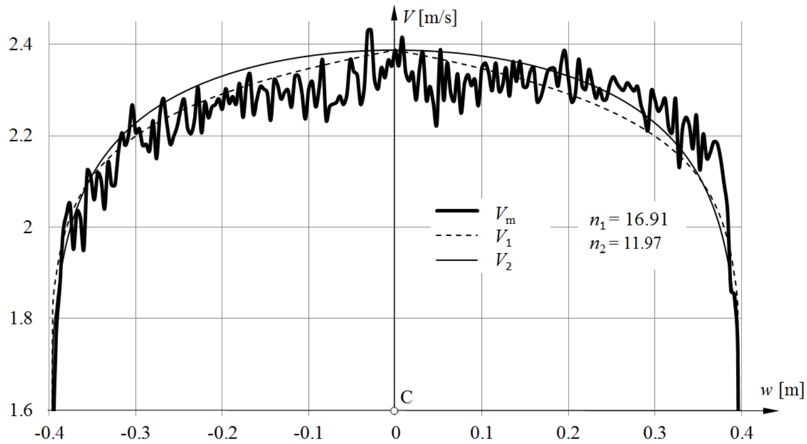

Fig. 5: Measurement results for above mentioned rounded rectangular duct.

The traversing step of the HWA anemometer probe (Fig. 6) was $\Delta w = \Delta h = 4\,\mathrm{mm}$. During the measurements, the same velocity V_C of the air flow at the point of intersection of the two axes should be provided and proved.

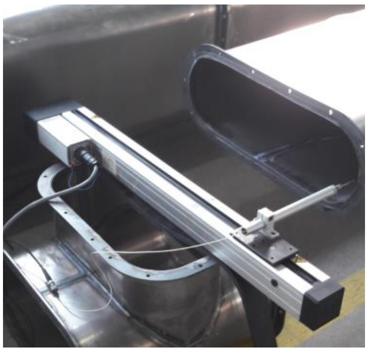

Fig. 6: Hot wire anemometer Dantec was used for velocity profile distribution measurement.

5. Conclusions

Both, the basic Prandtl power law for velocity distribution in circular ducts and its modified version presented in this paper, can be used for modelling of velocity distribution in ventilation rounded rectangular ducts. Basic Prandtl power law gives better results for circular cross-sections, whereas modified Prandtl power law gives better results for rectangular and rounded rectangular cross-sections.

At the present state of knowledge, determination of Prandtl power law exponent n requires measuring the velocity of flowing air in at least several points of the cross- section axis.

Acknowledgement

The authors would like to kindly thank the company for the performance of tested ducts and the delivery of air supply system. Authors also received institutional support BS 16/2013 granted by Faculty of Mechanical Engineering of UTP University.

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